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Algebra Note Card

Simplifying

Order of Operations

Ex: $3 \cdot 2^2 + 4 \div 4 - (1+2)^3 / 9$

Please:	**P**arenthesis	$3 \cdot 2^2 + 4 \div 4 - \underline{(1+2)}^3 / 9$
Excuse:	**E**xponents	$3 \cdot \underline{2^2} + 4 \div 4 - \underline{(3)^3} / 9$
My:	**M**ultiplication	$\underline{3 \cdot 4} + 4 \div 4 - (27)/9$
Dear:	**D**ivision	$12 + 4 \div 4 - (27)/9$
Aunt:	**A**ddition	$12 + 1 - 3$
Sally:	**S**ubtraction	$13 - 3 = 10$

Distributing : Multiply 3 by each term in the parenthesis

Ex. $3(x + 2y + 5) = 3x + 6y + 15$

Add Like Terms by grouping like terms together

Ex. $10x^2 - 4x^2 + 3x + 4x - 6 - 1 = 6x^2 + 7x - 7$

Radicals(√): Combine if each term has the same thing under the √

Ex. $2\sqrt{3} + 5\sqrt{3} = 7\sqrt{3}$ Not $2\sqrt{7} + 2\sqrt{3} \neq 4\sqrt{10}$

Solving One Variable Equations

Solving Equations: (3 Steps)

1st: Distribute

2nd: Combine Like Terms

3rd: Solve for x

Ex: $6(x - 6) - 4x = 2(3 - 2x) + 6$

$6x - 36 - 4x = 6 - 4x + 6$

$2x - 36 = 12 - 4x$

$+4x \quad\quad +4x$

$6x - 36 = 12$

$+36 \quad +36$

$6x = 48$

$\frac{6x}{6} = \frac{48}{6}$

$x = 8$

Solving Absolute Value Equations: Ex: $|x+3| = 5$

1st: Remove the | sign | by making two separate equations.

2nd: Solve each equation separately to find the two answers for x.

$x+3 = 5$ and $x + 3 = -5$

$-3 \quad -3$ and $-3 \quad -3$

$x = 2$ and $x = -8$

Solving Equations with Square Roots Ex: $\sqrt{x+3} = 4$

Square both sides: $\left(\sqrt{x+3}\right)^2 = 4^2$

The squared sign (^{2}) cancels with the square root sign (√)

$x + 3 = 16$

$x = 13$

Proportions

A **Ratio** is a comparison between two things.
Ex: The ratio of wheels to bikes is 2 to 1. Written as: $\frac{2 \text{ Wheels}}{1 \text{ Bike}}$

Proportions are built from ratios. Ex: A proportion can be used to find how many wheels are on 34 bikes.

Solve a proportion by **cross multiplying**

$\frac{2 \text{ Wheels}}{1 \text{ Bike}} = \frac{x \text{ Wheels}}{34 \text{ Bikes}}$

1st: Multiply the top of the first fraction by the bottom of the second fraction (2)•(34)

2nd: Multiply the bottom of the first fraction by the top of the second fraction (1)•(x)

$(1)\cdot(x) = (2)\cdot(34)$

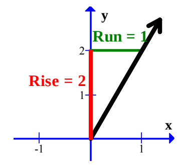
$x = 68$ Bikes

Slope

The **Slope** of a line is calculated as Rise over Run written $\frac{Rise}{Run}$

Ex: This line rises 2 units for every 1 unit it moves to the right. The slope of this line is 2

$\frac{Rise}{Run} = \frac{2}{1} = 2$



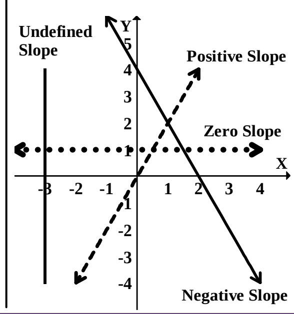
The Slope Formula

Helps find the slope from two points (x_1,y_1) and (x_2,y_2) $\frac{y_2 - y_1}{x_2 - x_1}$

Ex: Calculate the slope between points (1,0) and (2,3)

$\frac{y_2 - y_1}{x_2 - x_1} \quad \frac{3-0}{2-1} = \frac{3}{1} \quad slope = 3$

Examples of Slope



Graphing Lines

Slope Intercept Form

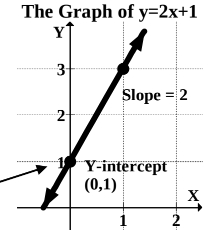
y = slope (x) + y-intercept

Example: y = 2x+1

The ***slope*** is 2. It is the # before the x

The ***y intercept*** is 1. It is the place where the line crosses the y axis.

The Graph of y=2x+1



Point Slope Equation

$y - y_1 = slope(x - x_1)$

Example: Find an equation with slope 2 going through point (1,3)

$y - 3 = 2(x - 1)$

$y - 3 = 2x - 2$

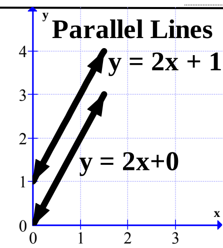
$+3 \quad\quad +3$

$y = 2x + 1$

Parallel Lines

Have the same slope

Ex: y = 2x + 1 and y = 2x + 0 both have slope = 2

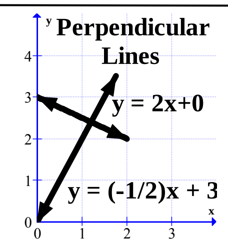


Perpendicular Lines

Have slopes which are negative reciprocals.

Ex: A line has slope $=\frac{2}{1}$

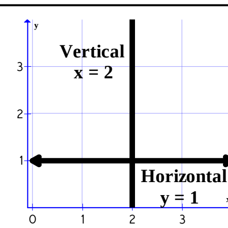
A line perpendicular has slope $-\left(\frac{2}{1}\right) = -\frac{1}{2}$



Vertical and Horizontal Lines have special equations:

Ex 1: x=2 is a Vertical line

Ex 2: y=1 is a Horizontal line



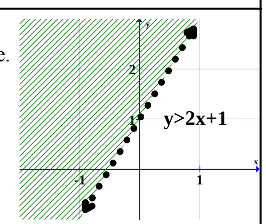
Graphing Inequalities

If y is > then shade above.

Ex: y > 2x + 1

Step 1: Graph y=2x+1 as a dotted line

Step 2: Shade the area above the dotted line.



Compound Inequalities

Compound Inequalities involve solving two separate problems which lead to a two part answer.

Ex: Solve for x: $-2 > 2x - 8 > 10$

1st: Write as two separate problems: $-2 > 2x - 8$ and $2x - 8 > 10$

2nd: Solve each problem separately

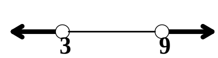
$-2 > 2x - 8 \quad\quad 2x - 8 > 10$

$+8 \quad\quad +8 \quad\quad +8 \quad +8$

$\frac{6}{2} > \frac{2x}{2} \quad\quad \frac{2x}{2} > \frac{18}{2}$

3rd: x is less than 3 or greater than 9: $3 > x \quad\quad x > 9$

4th: plot on a number line



Functions

Function: This graph is a function because every x has just one y value. Ex: when x =1, y = 3

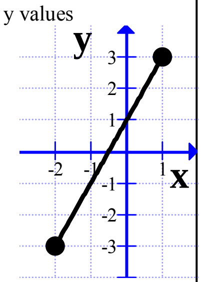
Domain means all of the possible x values

Range means all of the possible y values

Ex: This Function has:

Domain: $-2 \leq x \leq 1$

Range $-3 \leq y \leq 3$



Function Notation

Ex: Find f(3) for the function f(x) = 2x+1

f(3) means replace x with 3.

f(3) = 2(3) +1 = 7

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Algebra Note Card

Solving Systems Of Equations

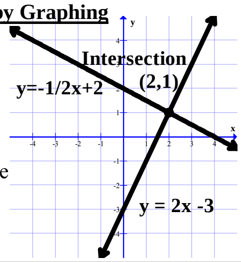
Solving Systems by Graphing

Ex $y = -1/2x + 2$
$y = 2x - 3$

1st: Graph the line
$y = -1/2x + 2$

2nd: Graph the line
$Y=2x-3$

Lines hit at (2,1)



Solving Systems by Substitution

Ex: Solve for x and y: $2x+y = 15$
$y = 3x$

1st: Substitute 3x into the first equation.

$2x + y = 15$
$2x+3x = 15$
$5x = 15$
$x = 3$

2nd: Plug x = 3 into the second equation to find y:
$y = 3x$
$y = 3(3) = 9$

The solution is the point (3,9)

Solving Systems by Linear Combination

Ex: Solve the two equations for x and y
$2x+5y = 16$
$-2x +6y = 6$

1st: Add up the two equations to cancel the x's and find y.

$2x+5y = 16$
$\underline{-2x +6y = 6}$
$11y = 22$
$y = 2$

2nd: Plug y = 2 into the first equation
$2x+5y = 16$
$2x+5(2)= 16$
$2x + 10 = 16$
$\underline{-10 \quad -10}$
$2x = 6$
$x = 3$

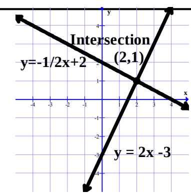
The Solution is (3,2)

3 Types of Solutions

Intersecting: Two lines intersect if their slopes are different. The lines intersect at one point.

Ex: $y=-\frac{1}{2}x+2$ *slope is* $-\frac{1}{2}$
$y=2x-3$ *slope is* 2

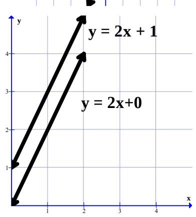
The solution is the intersection at (2,1)



Inconsistent: Two lines are inconsistent (do not intersect) if their slopes are the same but their y intercepts are different.

Ex. $y=2x+0$ $y-intercept\ is\ 0$
$y=2x+1$ $y-intercept\ is\ 1$

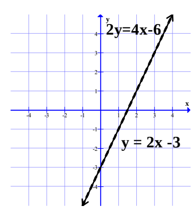
These two lines have the same slope of 2 but no solution. (intersection)



Equivalent Lines: Two lines are equivalent if their slopes and y intercepts are the same. Because the lines are the same, they intersect in an infinite number of points. In this example y = 2x-3 and 2y = 4x – 6 are the same line.

To see why divide both sides by 2. $\frac{2y}{2} = \frac{4x-6}{2}$

We find the equations are the same. $y = 2x - 3$



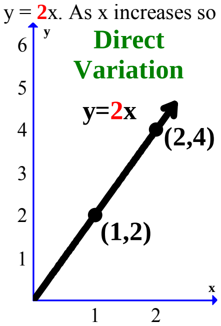
Direct and Inverse Variation

Direct Variation

Exists when an increase in one variable results in an increase in the other.

Ex: In the equation y = 2x. As x increases so does y.

The number 2 is called the constant of variation.

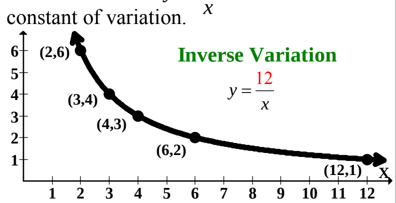


Inverse Variation

Exists when an increase in one variable results in a decrease in the other.

Ex: In the equation $y=\frac{12}{x}$, as x increases y decreases. When x = 2, y = 6 but when x = 12, y = 1.

The number 12 in $y=\frac{12}{x}$ is called the constant of variation.



Multiplying Binomials

The FOIL Method

Ex: $(x+3)(x+2)$

First	$(\underline{x}+3)(\underline{x}+2)$	$= x^2$
Outside	$(\underline{x}+3)(x+\underline{2})$	$= 2x$
Inside	$(x+\underline{3})(\underline{x}+2)$	$= 3x$
Last	$(x+\underline{3})(x+\underline{2})$	$= 6$

$x^2+2x+3x+6$ makes x^2+5x+6

Exponents

Multiplying: $x^3 \bullet x^4 = x^7$

Power to a Power: $(x^3)^4 = x^{12}$

Negative Powers: $x^{-2} = \frac{1}{x^2}$

Fractional Powers: $x^{\frac{1}{2}} = \sqrt{x}$

Zero Power: $x^0 = 1$

Simplifying with powers:
Ex.
$\frac{10x^5y^8}{2x^6y^2} = \frac{5y^6}{x}$

Solving Quadratic Equations

Solving Quadratics by Factoring

Ex: Solve for x: $x^2+5x +6 = 0$

1st: Factor $x^2+5x +6$ into $(x+3)(x+2)$

	x	+ 3
x	x^2	3x
+ 2	2x	6

2nd: Rewrite the original problem as $(x+3)(x+2) = 0$

3rd: Set each parenthesis equal to 0 and solve for the two answers.
$(x+3) = 0$ and $(x+2)=0$
$x=-3$ $x=-2$

Quadratic Formula

$$\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

a,b,c are the coefficients of the equation: $\mathbf{a}x^2 + \mathbf{b}x + \mathbf{c} = 0$

Ex: Solve $x^2 +5x + 6 = 0$
a= 1 b=5 c=6

$$\frac{-5\pm\sqrt{5^2-4(1)(6)}}{2(1)} = \frac{-5\pm\sqrt{1}}{2}$$

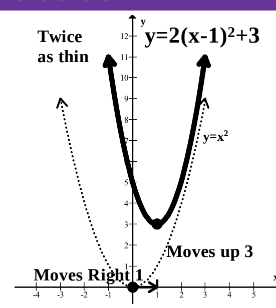
$$\frac{-5+1}{2}=-2 \quad or \quad \frac{-5-1}{2}=-3$$

There are two answers for x: x = -2 and x = –3

Graphing Parabolas

Ex: Graph: $y = 2(x-1)^2 + 3$
Compared to $y=x^2$

- The 1 moves the vertex 1 unit right
- The 3 moves the vertex up 3 units
- The 2 makes the graph twice as thin



Rational Expressions and Polynomials

Simplifying Rational Expressions

Factor out $2x^2$ $\frac{2x^3-10x^2}{2x^2}$

Cancel $2x^2$ on top and bottom $\frac{2x^2(x-5)}{2x^2}$

Rewrite as x-5 $\frac{(x-5)}{1} = x-5$

Degree	Type	Example
0	Constant	3
1	Linear	x+3
2	Quadratic	x^2+3x+4
3	Cubic	x^3+2x^2+4x

Solving Rational Equations

Ex: $\frac{2}{x}+\frac{1}{3}=\frac{1}{2}$

1st: Find the Lowest Common Denominator (LCD)

LCD is 6x

2nd: Multiply each term by the LCD

$\frac{(6x)2}{x}+\frac{(6x)1}{3}=\frac{(6x)1}{2}$

3rd: Cancel to eliminate each fraction

$\frac{12x}{x}+\frac{6x}{3}=\frac{6x}{2}$

4th Solve like a regular problem

$12+2x=3x$

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