

### Decimals

#### Adding Decimals

Ex: Add  $2.3 + 45.06$

**Step 1:** Line up the decimal points

$$\begin{array}{r} 2.3 \\ + 45.06 \\ \hline 2.30 \\ + 45.06 \\ \hline = 47.36 \end{array}$$

**Step 2:** Put a "0" on the right of the 2.3

**Step 3:** Add from the top down

#### Multiplying Decimals

Ex: Multiply  $1.25 \times 3.2$

**Step 1:** Count the number of decimal places to the right of the decimal for each number.

1.25 has two numbers to the right of the decimal (the numbers are 2 and 5)

3.2 has one number to the right of the decimal (the number is 2)

**Step 2:** Add the total number of places to the right of the decimal from step 1.

2 plus 1 = 3 decimal places

$$\begin{array}{r} 1.25 \\ \times 3.2 \\ \hline 250 \\ 3750 \\ \hline 4000 \end{array}$$

**Step 3:** Multiply the original numbers without their decimals  $125 \times 32$

**Step 4:** Take your answer of 4000 and move the decimal 3 places to the left (The 3 places are from step 2 above) to get

4.000 or just 4

#### Dividing Decimals

Ex: Divide  $2.25 \overline{)13.5}$

**Step 1:** Determine whether 2.25 or 13.5 has more numbers to the right of the decimal. (In this case 2.25 has two places)

**Step 2:** Move the decimal 2 places to the right for both numbers.

2.25 becomes 225  
13.5 becomes 1350 (notice the added 0)

$$\begin{array}{r} 6 \\ 225 \overline{)1350} \\ \underline{1350} \\ 0 \end{array}$$

**Step 3:** Divide 225 into 1350

**Step 4:** The answer is just 6

#### Fractions to decimals

Ex: Change  $\frac{3}{4}$  into a decimal

**Step 1:** Divide 3 by 4. Add 2 zeros to the right of the 3 and put a decimal above the existing 3.00

$$\begin{array}{r} .75 \\ 4 \overline{)3.00} \\ \underline{280} \\ 200 \\ \underline{200} \\ 0 \end{array}$$

**Step 2:** Divide 4 into 300

**Step 3:** Your answer as a decimal is .75

#### Multiplying Large Numbers by Decimals

Ex:  $25,000 \times .04$

**Step 1:** For the large number put a decimal at the end 25,000., and count the number of decimal places to the left of the decimal needed to remove the zeros. In this case +3 places (25,000.)

**Step 2:** For the small number count the number of decimal places to the right of the decimal. There are 2 places (.04)

Which we will note as -2 because we counted to the right

**Step 3:** Note the balance of +3 (from step 1) and -2 from step 2 makes:  $3 - 2 = 1$  place

**Step 4:** Remove the zeros at the end of the original 25,000 to leave just 25

Remove the two decimal places to the right of the decimal in .04 to leave just 4

**Step 5:** Multiply  $25 \times 4 = 100$ . Add 1 zero because the balance in step 3 above was 1 decimal place.

So 100 becomes 1,000

#### Scientific Notation

Displayed as one number to the left of the decimal times a power of 10

Ex:  $3.4 \times 10^3$  is in Scientific Notation  $10^3$  moves the decimal 3 places to 3,400

Ex:  $2.1 \times 10^{-2}$  is in Scientific Notation  $10^{-2}$  move the decimal 2 places left = .021

#### To Multiply in Scientific Notation

Ex:  $3.4 \times 10^3$  times  $2.1 \times 10^{-2}$

**Step 1:** Multiply the  $3.4 \times 2.1 = 7.14$

**Step 2:** Add the powers  $10^3$  and  $10^{-2}$   $3 - 2 = 1$  so  $10^1$

**Step 3:** Put step 1 and 2 together:

Solution  $7.14 \times 10^1$

### Number Theory

**Prime Numbers** can only be divided by themselves and 1. Ex: 2,3,5,7,11,13...

**Prime Factoring** involves rewriting a number as the product of prime numbers.

Examples	Prime Factorization
15	$5 \cdot 3$
18	$2 \cdot 3 \cdot 3$
100	$2 \cdot 2 \cdot 5 \cdot 5$

**The Greatest Common Factor (GCF)** of two numbers is the largest number that divides into both numbers.

Examples	Greatest Common Factor
10 and 5	5
18 and 12	6
24 and 36	12

#### Finding the Greatest Common Factor

Multiply together the prime factors the two numbers have in common:

Ex: Find the Greatest Common Factor of 30 and 105

**Step 1:** Write the prime factors of 30  $30 = 2 \cdot 3 \cdot 5$

**Step 2:** Write the prime factors of 105  $105 = 3 \cdot 5 \cdot 7$

**Step 3:** Both numbers have a 3 and a 5 as their prime factors. The Greatest Common Factor is  $3 \cdot 5 = 15$

#### Least Common Multiple (LCM)

The smallest number that two numbers divide into. (Ex: the LCM of 2 and 3 is 6)

Ex: How to find the LCM of 12 and 18

**Step 1:** Write out the prime factors of each number

$$12 = 2 \cdot 2 \cdot 3$$

$$18 = 2 \cdot 3 \cdot 3$$

**Step 2:** Select the minimum factors that both number require. LCD is  $2 \cdot 2 \cdot 3 \cdot 3 = 36$

### Fractions

#### Multiplying Fractions

$$\frac{\text{top}}{\text{bottom}} \cdot \frac{\text{top}}{\text{bottom}} \quad \text{Ex: } \frac{3}{4} \cdot \frac{1}{2} = \frac{3 \cdot 1}{4 \cdot 2} = \frac{3}{8}$$

To find the **Reciprocal** of a fraction just flip the fraction over.

Ex: Reciprocal of  $\frac{3}{4}$  is  $\frac{4}{3}$

#### Dividing Fractions

Flip the second fraction over and multiply

$$\text{Ex: } \frac{3}{5} \div \frac{1}{2} = \frac{3}{5} \cdot \frac{2}{1} = \frac{3 \cdot 2}{5 \cdot 1} = \frac{6}{5}$$

#### Adding Fractions

To add, both fractions must first have the same denominator. Then add numerators.

Ex: Add  $\frac{5}{18}$  and  $\frac{1}{12}$

**Step 1:** Find the Lowest Common Denominator which is 36. (See Lowest Common Denominator)

$$\text{Step 2: Change each fraction into } 36^{\text{th}} \text{ denominator}$$

$$\frac{(2) \cdot 5}{(2) \cdot 18} = \frac{10}{36} \quad \frac{(3) \cdot 1}{(3) \cdot 12} = \frac{3}{36}$$

**Step 3:** Add the numerators (tops) not the denominators (bottoms)

$$\frac{10}{36} + \frac{3}{36} = \frac{13}{36}$$

Answer is:  $\frac{13}{36}$

#### The Lowest Common Denominator

is the smallest denominator that the bottom of both numbers divide into.

Ex 1: The LCM of  $\frac{1}{2}$  and  $\frac{1}{3}$  is  $\frac{6}{6}$

Ex 2: Find the LCM of  $\frac{5}{18}$  and  $\frac{1}{12}$

**Step 1:** Look only at the denominators (bottoms) of the fractions. Find the least common multiple of 18 and 12.

The smallest number 18 and 12 both divide into is 36 (See Least Common Multiple above)

The Least Common Denominator is 36

#### Reducing Fractions

Ex: Reduce the fraction  $\frac{30}{42}$

**Step 1:** Rewrite the numerator (top) using prime factors:  $30 = 2 \cdot 3 \cdot 5$

**Step 2:** Rewrite the denominator (bottom) using prime factors:  $42 = 2 \cdot 3 \cdot 7$

**Step 3:** Rewrite the fraction using the factors. Cancel factors that appear on the top and bottom.

$$\frac{30}{42} = \frac{2 \cdot 3 \cdot 5}{2 \cdot 3 \cdot 7} = \frac{5}{7}$$

### Working with Numbers

**Real Numbers:** Are any numbers that can be written as a decimal including non-repeating decimals.

Examples: 5.4512693... or  $\frac{1}{3} = .333$  or 7

**Rational Numbers:** Any number that can be written as a fraction or repeating decimal.

Examples:  $\frac{1}{3} = .333...$  or .314314... or 8

**Irrational number:** A number that can not be written as a fraction nor as a repeating or terminating decimal.

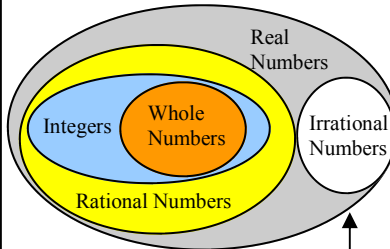
Examples:  $\sqrt{3} = 1.73205...$  or  $\pi = 3.1415...$

**Integers:** Are positive and negative whole numbers.

Examples: ...-3, -2, -1, 0, 1, 2, 3...

**Whole Numbers:** Are positive integers.

Examples: 1, 2, 3...



#### Relationship Between Numbers

Ex: An integer is a rational number and a real number but not a whole number.

#### Plotting on a number line

Ex: Plot  $x \leq 4$

The **Absolute Value** is the distance a number lies from 0 on the number line.

Positive numbers remain positive.  $|5| = 5$

Negative numbers become positive.  $|-3| = 3$

### Simplifying

#### Order of Operations

Ex:  $3 \cdot 2^2 + 4 \div 4 - (1 + 2)^3 / 9$

Please: Parenthesis

$3 \cdot 2^2 + 4 \div 4 - (1 + 2)^3 / 9$

Excuse: Exponents

$3 \cdot 2^2 + 4 \div 4 - (3)^3 / 9$

My: Multiplication

$3 \cdot 4 + 4 \div 4 - (27)/9$

Dear: Division

$12 + 4 \div 4 - (27)/9$

Aunt: Addition

$12 + 1 - 3$

Sally: Subtraction

$13 - 3 = 10$

**Distributing:** Multiply 3 by each term in the parenthesis

Ex:  $3(x + 2y + 5) = 3x + 6y + 15$

**Add Like Terms** by grouping like terms together

Ex:  $10x^2 - 4x^2 + 3x + 4x - 6 - 1 = 6x^2 + 7x - 7$

**Multiplying Negatives and Positives**

- · - = +    + · - = -  
+ · + = +    - · + = -

### Algebraic Equations

#### Solving Problems by Subtracting

Subtract from both sides of the equation to get x by itself

Ex:  $x + 8 = 12$   
 $\frac{-8 \quad -8}{x = 4}$

#### Solving Problems by Dividing

Divide both sides of the equation to get x by itself

Ex:  $\frac{3x}{3} = \frac{12}{3}$   
 $x = 4$

#### Solving Two Step Equations

Ex:  $3x + 7 = 34$   
First subtract 7  $\rightarrow \frac{-7 \quad -7}{3x = 27}$   
then divide by 3  $\rightarrow \frac{3x}{3} = \frac{27}{3}$   
 $x = 9$

#### Solving Problems by Adding

Add to both sides of the equation to get x by itself

Ex:  $x - 3 = 12$   
 $\frac{+3 \quad +3}{x = 15}$

#### Solving Problems by Multiplying

Ex: Solve for x:  $\frac{1}{4}x = 7$

**Step 1:** Locate the fraction in front of the variable. In this case  $(\frac{1}{4})$

**Step 2:** Multiply both sides by the reciprocal of this fraction.  $(\frac{4}{1})$   
Ex:  $\frac{4}{1} \cdot \frac{1}{4}x = (\frac{4}{1}) \cdot 7$   
 $x = (\frac{4}{1})(\frac{7}{1})$

**Step 3:** Cancel the 4's on the left side to leave just x. Multiply the right side to get 28.  
 $x = \frac{28}{1} = 28$

#### Solving Problems with Variable on Both Sides

Ex:  $2x - 36 = 12 - 4x$

**1st:** Move x's to one side.

(In this case we chose the left side)

$2x - 36 = 12 - 4x$   
 $\frac{+4x \quad +4x}{6x - 36 = 12}$

**2nd:** Move # to the other side (The right side)

$\frac{+36 \quad +36}{6x = 48}$

**3rd:** Divide by 6

$\frac{6x}{6} = \frac{48}{6}$   
 $x = 8$

#### Checking Your Solution

Plug the number 8 back into the original equation to see if it works.

$2x - 36 = 12 - 4x$   
 $2(8) - 36 = 12 - 4(8)$   
 $16 - 36 = 12 - 32$   
 $-20 = -20 \quad \odot$

### Area and Perimeter

#### Triangle

$Area = \frac{1}{2} B \cdot H$



Perimeter = Distance Around

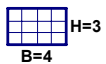


Perimeter = 5 + 5 + 6 = 16

$Area = \frac{1}{2} \cdot 6 \cdot 4 = 12$

#### Rectangle

$Area = B \cdot H$



Perimeter = Distance Around



Perimeter = 4 + 3 + 4 + 3 = 14

$Area = 4 \cdot 3 = 12$

#### Circle

$Area = \pi r^2$



Circumference =  $2\pi r$

$= 2\pi \cdot 3$

$= 6\pi$

$Area = \pi \cdot 3^2$

$Area = \pi \cdot 9$

$Area = 3.14 \cdot 9 \approx 28.26$

$\approx 6 \cdot (3.14) \approx 18.84$

### Powers

**Squaring:** Ex:  $5^2$  means  $5 \cdot 5 = 25$

**Raising to a Power:** Ex:  $2^4$  means  $2 \cdot 2 \cdot 2 \cdot 2 = 16$

**Square Root:** Ex:  $\sqrt{9}$  means what number times itself equals 9. The answer is 3.

### Coordinates

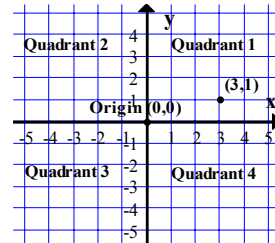
The **Origin** is at the point (0,0)

The four **Quadrants** are labeled

The **x-axis** goes from left to right

The **y-axis** goes up and down

**Coordinate Form** of a point (x,y). The first number shows how far the point lies to the right (or left) of the origin. The second number shows how far above (or below) the origin.



#### Plotting a Point

Ex: Plot (3,1)

Step 1: Start at (0,0)

Step 2: Move 3 units right

Step 3: Move 1 unit up and mark it with a dot.

### Percent

**Percent** means per 100. Ex: 20% means  $\frac{20}{100}$

#### Changing a Fraction to a Percent

Ex: Change  $\frac{3}{4}$  to a percent.

**Step 1:** Rewrite as a fraction over 100 by multiplying by 25  $\frac{3(25)}{4(25)} = \frac{75}{100}$

**Step 2:** The numerator 75 is the % = 75%

#### Changing a Percent to a Fraction

Ex: Change 4% to a fraction

Write the % over 100 then reduce  $\frac{4}{100} = \frac{1}{25}$

### Tables, Graphs and Equations

An equation like  $y = 2x$  can also be shown as a **graph** or as a **table**

Table for  $y = 2x$

X	Y
1	2
2	4
3	6
4	8
x	2x

**Graph of  $y = 2x$**

